SWEDISH ECONOMIC POLICY REVIEW 3 (1996) 203–232

Reducing the Cost of Government Debt: The Italian Experience and the Role of Indexed Bonds

Silverio Foresi,* Alessandro Penati**
and George Pennacchi***

Summary

■ The pricing of government debt in a number of financial markets is analyzed to see how high-debt countries should manage their liabilities. We focus on the Italian experience and also consider whether indexed bonds can reduce the cost of funding the public deficit. In particular, we attempt to quantify the risk premia that investors require on different forms of government debt. Our approach is market oriented, based on government debt prices observed in the market place. Due to practical considerations, the nature of our analysis is partial equilibrium, rather than general equilibrium. We try to understand how investors price treasury securities and attempt to measure risk premia due to inflation and real interest rates on various forms of debt. Our findings suggest that a government can reduce the cost of debt by increasing its market depth and liquidity and by avoiding financial innovations as a debt management policy. Finally, our analysis suggests that real indexed bonds can reduce the cost of debt, but that the savings for a government can be seriously misjudged by an overly simple analysis. ■

**Silverio Foresi is Assistant Professor of Finance at the Stern School of Finance, New York University.*

***Alessandro Penati is Professor of Economics at the Department of Economics, Università di Sassari.*

****George Pennacchi is Associate Professor of Finance at the Department of Finance, University of Illinois at Urbana-Champaign.*

SWEDISH ECONOMIC POLICY REVIEW 3 (1996) 203–232

Reducing the Cost of Government Debt: The Italian Experience and the Role of Indexed Bonds

Silverio Foresi, Alessandro Penati and George Pennacchi*

At the end of 1994, the stock of Italian Government debt reached the level of 1,937,069 billion lire, or 117.2 per cent of GDP. If this debt is calculated according to the criterion defined in the Maastricht Treaty, its proportion of GDP increases to 124.5 per cent. During the same year, the budget deficit , net of privatization proceeds, reached 155,900 billion lire, some 9.4 per cent of domestic product. Interest payments alone amounted to 10.5 per cent of GDP, as the country experienced a primary surplus. The value of the Government debt issued was 100.6 per cent of the budget deficit; 92.5 per cent of the debt placed by the Treasury (net of refunding) was in the form of medium and long term bonds, with the remainder funded mostly by means of Treasury bills.

Debt management policy is a substantive policy issue in Italy. Italian Government debt is issued mainly in the domestic market and denominated in lire. Foreign debt accounts for only 4 per cent of the total stock, while ECU indexed bonds, which are auctioned to domestic investors, have practically ceased to be issued after Italy's exit from the European Monetary System and never exceeded 3 per cent of the yearly Treasury funding program. Floating rate bonds (*Certificati di Credito del Tesoro*, CCT), currently 7 year notes with a 30 basis point spread over the 6 month Treasury bill rate, account for roughly 35% of the lira denominated Government debt. Treasury bills (*Buoni Ordinari del Tesoro*, BOT), sold at regular bi-weekly auctions for maturities of 3, 6 and 12 months, account for another 27%. Fixed coupon bonds (*Buoni del Tesoro Polien-*

* *We thank Lars Svensson, Nils Gottfries and Anders Vredin for their comments. Stefano Nannizzi, Jason Karceski, and Manlio Marchesi provided helpful research assistance. Analysis s.p.a. provided part of the Italian data.*

nali, BTP), which are also auctioned regularly twice a month for maturities of 3, 5, 7, 10 and 30 years, represent 33% of the total. The remaining debt consists of bonds with an implied option to extend the maturity of the bond (*Certificati di Credito con Opzione*, CTO), which ceased to be issued in 1992, and a new 2 year zero coupon bond (*Certificato di Credito Zero*, CTZ) recently launched by the Treasury.

The composition of the stock of debt does not reflect the current debt policy of the Government. Until 1988, floaters and Treasury bills were the only source of Treasury funding, as the inflation rate risk premium was deemed to be too high for fixed coupon bonds. It was not until the end of 1988 that the first liquid 4 year straight bond was successfully placed with investors. At the same time the Treasury tried to lengthen the maturity of the debt by issuing CTOs, which are 3 and 4 year fixed coupon bonds with an option to extend the life of the securities by an equal number of years. The narrowing of the bands of fluctuations for the lira within the European Monetary System at the end of 1989 allowed the Government to issue a 6 year fixed coupon BTP in June 1990 and then a 10 year BTP in March 1991. The security enjoyed immediate liquidity, especially when the LIFFE exchange in London launched a BTP futures contract in the Fall of that year and, one year later, when a similar contract was traded on the Milan electronic futures market, MIF. In mid-1993, the first issue of a 30 year fixed coupon bond took place. Meanwhile, the liquidity of Italian Government debt greatly improved as a result of the introduction of primary dealers who guarantee two-way prices within narrow spreads and participate in Treasury auctions for a fixed fraction of the debt being issued. The creation of additional derivatives, including a futures contract on 3 month Eurolire deposits, options on futures contracts, and interest rate swaps, made hedging interest rate risk easier. As a result of these changes, the Treasury has relied on straight bonds (BTPs) to fund approximately two-thirds of the public sector deficit since 1991.

What conclusions can be drawn from the Italian experience? One way to judge the management of Italy's debt would be to search for testable implications from the large and interesting literature on optimal debt policy. Gale (1994) considers the welfare aspects of government debt policy, focusing on the risk-sharing possibilities of government debt issue. His work extends previous analysis by Fischer (1983), Peled (1985), and Bohn (1988). When private security markets are incomplete, the potential for non-neutral, welfare-improving government intervention arises.

One area where markets may be incomplete is in the provision of riskless securities. Private markets may be unable to create a sufficient supply of riskless assets because of credit risk – a risk that a government's power of taxation might overcome. Alternatively, markets may be incomplete because private financial intermediaries or instruments which enable intergenerational trade may not exist (though private pension funds may be an example of this sort of private institution). Government provision of previously unavailable riskless securities or long-maturity claims can improve welfare by completing markets, thereby expanding investors' risk-sharing opportunities. When markets are incomplete due to differences in investor information, Gorton and Pennacchi (1990) show that government creation of riskless securities can also improve welfare by enhancing liquidity. Government securities, such as treasury bills, can provide lesser-informed investors with a lower cost transactions vehicle, thereby reducing their costs of trading.

As Fischer (1983) points out, government debt policy is usually, if not always, only one of several ways of improving welfare when markets are incomplete. If a government can directly set taxes and make transfers, there is no need for an interventionist debt policy. One area where debt policy may be uniquely important is in its effect on the time consistency and credibility of government fiscal policies. For example, a long-run optimal policy for a government may be to smooth current taxation relative to current government spending, running fiscal deficits during times of extraordinary government spending requirements (e.g., military or economic crises) and running fiscal surpluses during times of relative prosperity. However, following a period of deficits and an increase in the stock of government debt, the government may be tempted to deviate from this long-run optimal policy by reducing the payments it makes to existing government bondholders; such a policy is optimal in the short run, but not consistent with the long-run optimal policy. This payment reduction could occur via outright default or an unexpected increase in the taxation of bondholders' returns (either directly or through an inflation tax). Recognizing this possibility, rational investors in government debt will initially demand a premium commensurate with the likelihood of the government's deviation. Importantly, the likelihood that a future government reneges on debt repayments may depend on the form of the debt that it inherits. For example, if the mechanism by which a government can deviate is via an unexpected increase in inflation, this mechanism will only be effective and, therefore, will only be carried out if the existing debt has long maturity and nominal coupons, rather than being of short maturity or indexed to the cost of living.

This paper takes a market-oriented approach to measuring the risk premia that investors require on various forms of government debt. Out of practical necessity, the analysis is partial, rather than general equilibrium: the current quantity and composition of debt is taken as given. The approach is also positive, rather than normative. Any policy implication that might result from the analysis should thus be treated with caution.

1. Do floaters help?

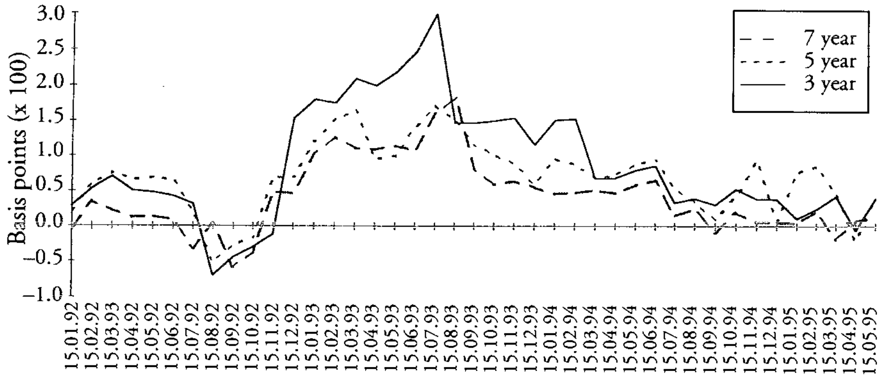
Throughout the 1980s, the vast majority of the debt issued by the Italian Government, excluding Treasury bills, was in the form of floaters, called CCT (*Certificati di Credito del Tesoro*). It is only since 1991, when the first 10 year bond was issued, that fixed coupon Treasury securities became the primary source of Government funding. Still, roughly one-third of the current stock of debt is composed of CCTs. There are two reasons why a high-debt country would like to issue floating rate notes. Both of them have been behind the Italian Government debt policy. First, because coupons are tied to short-term interest rates, floaters may provide a good hedge against inflation if short rates are freely set in the market place and reflect investors' expectations of inflation. Even though they do not provide a perfect hedge, they nonetheless allow a government to borrow long-term funds, presumably without paying an inflation risk premium.

Second, they might reduce the probability of a default, and thus the default premium associated with it. The literature on default, such as Alesina, Prati and Tabellini (1990), shows that a default on government debt could be a rational, market-equilibrium event. A government could be forced into default if investors were to ration the treasury by refusing to lend it additional money at any price, thereby halting its ability to roll over existing debt. This could occur if investors believe that the government may be unable to raise taxes to repay its debt following a negative shock to the economy. As in the standard rationing model, the borrower would face a negatively upward sloping supply of funds. The probability of default is minimized, the fewer the number of times the government accesses the credit market, and the smaller the amount of money borrowed each time. A floater is then equivalent to a strategy of rolling over short-term debt, but with the advantage of a longer debt maturity, thereby reducing default risk.

By examining the data, it appears that little, if anything, was gained by issuing floaters; CCTs turned out to be even more expensive than fixed coupon bonds. We assessed the relative value of CCTs in two different ways. First, since these floaters pay a coupon twice a year, a standard benchmark is the 6 month LIBOR rate on Eurolira deposits. The LIBOR, rather than the BOT rate, is used because the arbitrage of floaters occurs through the swap market, which is tied to Eurolira rates. However, until February 1995, floater coupons were indexed to the 12 month Treasury bill rate, thereby introducing a mismatch between the maturity of the base rate and the coupon reset interval. In addition, each coupon was based on the average yield prevailing in Treasury bill auctions during the second and the third month preceding the coupon reset date. Thus, basis swaps between Treasury bill rates and Eurocurrency rates, though available, could not be used for an exact arbitrage between floaters and money market rates. Nonetheless, the coupon spread over the LIBOR rate, measured as the current floater coupon divided by the floater price (this division was necessary since floaters are often traded below par), remains a valid indication of the relative cost of the debt instrument since 6 and 12 month Treasury bill rates are highly correlated and display little volatility. We limited the analysis to the period January 1992 to May 1995, because a liquid secondary market was in place, Eurolira and 10 year Government bond futures contracts were available, and international capital movements were completely liberalized. For the 15th day of each month, we selected the most liquid CCTs with a remaining life of 3, 5 and 7 years, respectively.

Figure 1 shows that the spreads over LIBOR are very large for every maturity, indicating that the Government continued to pay a substantial default risk premium. Only during the 1992 exchange rate crisis that led to the lira's exit from the European Monetary System does the spread turn negative. This was probably due to the Eurocurrency market's lack of liquidity during times of extreme currency turbulence which pushed up Eurolira rates, rather than a diminishing perception of default risk. Part of the spread may also reflect Italian commercial banks' lack of expertise in arbitraging CCTs with the money market, rather than a risk premium. The figure clearly indicates that 7 year floaters typically command a smaller spread than 3 year CCTs. The opposite is likely to be true if default risk were the only explanation. Seven year floaters are likely to be more fairly priced because the swap market is most liquid for this ma-

Figure 1. CCT floaters: spread over 6 month LIBOR



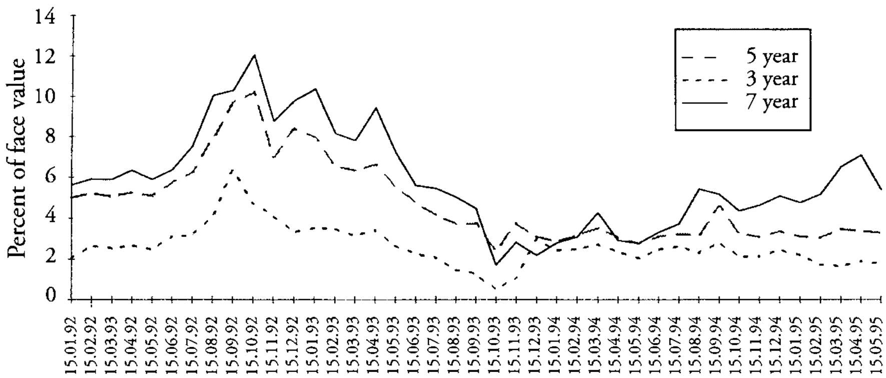
turity, thus making arbitrage relatively easier. A few months ago, the Treasury began to issue floaters with coupons tied to the 6 month Treasury bill and no indexation lag. This innovation should facilitate arbitrage activity; however, a spread for this new type of CCT can still be observed.

A second way to evaluate CCTs is to price them relative to the term structure of Treasury bonds. We followed the approach of El-Karoui and Geman (1992) where the price of a floater at any date is the discounted value, using the existing term structure, of the risk-adjusted expected coupons. Figure 2 shows the difference between the theoretical prices of the three CCTs and their market prices for the same period examined earlier[1]. Even though the difference has narrowed considerably since the end of 1993, it remains extremely large. This indicates that floaters were even more expensive than fixed coupon bonds. Because any default risk premium must necessarily be the same for any security issued by the Italian Treasury, and an inflation risk premium can only be smaller for floaters, the large discount can only be attributed to market segmentation and an excess supply of this type of Government debt.

The Italian experience with CCTs provides us with the following lessons. First, floaters do not necessarily reduce the cost of borrowing because of a lower default and inflation risk premium. Second, if arbitrage among various segments of the treasury debt market is not viable, supply

[1] The implied forward rates and the zero coupon bond term structure were obtained by fitting the Cox, Ingersoll and Ross (1985) model to the existing BTP prices, using a maximum likelihood approach.

Figure 2. CCT floaters: theoretical – market price



effects can push the effective cost of floaters even above that of equivalent-maturity, fixed-coupon bonds.

2. Debt management and financial innovation

A government might reduce its cost of funding by helping to complete financial markets. This can be done by issuing debt with payoffs that private financial institutions and markets are unable or unwilling to provide. By purchasing these debt securities, investors can obtain previously unavailable state-contingent cash flows that could lead to a better allocation of risk between investors and the government (i.e., current and future taxpayers). While, in principle, government financial innovation appears to be a promising avenue to debt management, the Italian experience has been disappointing. Between 1988 and 1990, the Italian Treasury found it virtually impossible to issue fixed-coupon bonds for maturities exceeding four years. Thus, it issued 3 and 4 year fixed-coupon bonds, called CTOs, which contained an embedded option to extend the maturity of the bond at the original conditions for an additional 3 or 4 year period. The bond was equivalent to a 3 (or 4) year bond plus a call option on a 6 (or 8) year bond that could be exercised in 3 (or 4) years; alternatively, the CTO was equivalent to a 6 (or 8) year straight bond, plus a put option that could be exercised after 3 (or 4) years. The Government's motive was to "test the water" by giving investors a long-term investment opportunity, but with insurance against its failure to bring down inflation.

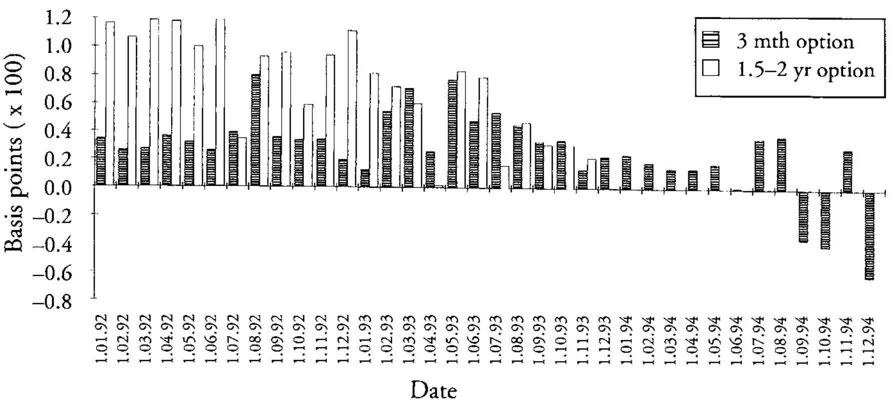
By issuing CTOs, the Treasury offered the market a previously unavailable long-term, low interest rate risk security. However, investors failed to appreciate the implied extendible option, as CTOs were usually traded as if they were simple 3 (or 4) year bonds; sometimes even at a relative discount. It is interesting to note that this mispricing remained even in 1992, when the Treasury was finally able to issue 10 year straight bonds, and futures as well as option contracts were available to hedge long-run interest rate risk. In that year the Government discontinued issuing CTOs.

Figure 3 shows the difference between the theoretically implied option price and its market value, under the assumption that the CTO's straight bond component was priced in line with the fitted term structure[2]. For every month, beginning in January 1992, the difference is calculated for two bonds, one having an implied option maturity of three months and one being the most liquid bond having an implied option maturity longer than 1.5 years. Data for the longer option were unavailable beyond 1993 since CTOs had not been issued in the previous year and a half. For simplicity, theoretical prices were calculated using the Black-Scholes model, which may overestimate prices of options having maturities that are long relative to the underlying bond, as the model does not consider the reduction in bond variance over time. To help correct for this bias, the bonds' estimated volatilities were adjusted downward by taking their means over the remaining life of the bond. In any case, the potential bias is negligible for the three-month option.

The figure shows that the market failed to pay the full theoretical price for these options, except for a few months in 1994 when it actually paid more. The mispricing was more pronounced for longer maturity options. Several conclusions might be drawn from this experience. First, financial innovation, per se, may not reduce government funding costs if there is insufficient demand for the new instruments. Rather than experimenting, a treasury might limit itself to issuing instruments for which a private market already exists, but for which the government has a natural comparative advantage as a low cost supplier. The U.S. experience with Treasury bond strips is a clear-cut example. The immunization needs of large institutional investors created a strong demand for zero-coupon bonds that was initially satisfied by private institutions selling strips from Treasury coupon bonds held in privately created trusts. The U.S. Treasury now satisfies this demand by allowing direct ownership of individual Treasury

[2] See footnote 1 for the methodology used.

Figure 3. CTO implied option: theoretical – market price



coupons and principal payments. Presumably, the Treasury now obtains the gains previously paid to the creators of private trusts. Second, the demand for government debt with embedded options will be enhanced if the underlying security already exists (allowing for easy hedging) and the options can be stripped and traded separately (allowing purchase by investors who may want the option but not the straight bond). None of these conditions existed for CTOs and this could explain why longer term implied option mispricing was always more pronounced than in the short term. Governments should keep these principles in mind when considering proposals for issuing treasury bonds containing long-term options on public-sector enterprise shares, as are currently being discussed in a number of countries.

3. Real indexed bonds: the Italian experience

It is often argued that real indexed bonds are an obvious solution for a high debt country. These bonds can reduce the inflation risk premium paid by the government because the incentive to levy an inflation tax is minimized. They may also signal a government's belief in lower inflation, which may reduce investors' inflationary expectations and aid the conduct of monetary policy, by indicating more accurate market expectations about inflation. Indexed bonds may also serve investor demands, especially institutions which manage retirement funds and wish to hedge the risk of long-term losses in purchasing power. Why, then, do we not see governments issuing indexed bonds on a large scale?

Italy issued only one indexed bond on August 1st, 1983, which matured ten years later. The bond took the form of a private placement. It paid an annual coupon of 2.5% that was indexed to the change in the value added deflator during the previous year. The principal amount was also indexed in the same way. The bond was not a success for several reasons. The issue was too small to be liquid and the Italian bond market, then an unregulated over-the-counter market, was not functioning well. The indexation had a one year lag, and the price index was unfamiliar to investors. The Government preferred the value added deflator to the consumer price index out of concern for imported inflation, as the lira's participation in the European Monetary System was not yet credible at that time.

We carried out an *ex-post* accounting exercise of the Government savings from issuing that bond. Of course, risk premia are *ex-ante* concepts. But a descriptive analysis can still provide some interesting information on the effective burden of the various forms of government debt. To reckon the *ex-post* burden, we capitalized the cash flows actually paid by the Government up to the day of maturity and compared them with those generated by alternative forms of borrowing. Table 1 (second column) shows the cash flows paid by the indexed bond. During this period, the growth rate of the value added deflator declined from 14.1% in 1983, to 4.9% in 1992. The consumer price index declined even faster, as the European Monetary System led to a real appreciation of the currency. The capitalized cash flows of the real bond would have been 266.5 lire for every 100 of principal amount at the end of the 10 year period[3]. Instead of the indexed bond, the Treasury could have chosen to issue a single floater, to roll over 1 year Treasury bills, or to issue a basket of securities with characteristics similar to the outstanding debt. The cash flows generated by these three alternatives, together with the differences from those of the indexed bonds, are also shown in Table 1. For the floater, we took a CCT that was issued on the same day as the indexed bond. Because it had a maturity of only five years, we rolled it over in 1988 with an identical floater that expired on August 1st, 1993. The cash flows after 1986 were calculated net of the witholding tax that was introduced in that year. For the Treasury bills, we rolled over a 12 month security, with the yield at the auction closest to the first day of August in each year. For the basket

[3] All the cash flows in Table 1 are capitalized until the index bond redemption date by using yields for 6 month BOT.

Table 1. Ex-post cost analysis of the Italian indexed bond

Date	Indexed bond (deflator)	Indexed bond (CPI)	Cash flow diff	CCT (1.08.1983 1.08.1988)	Cash flow diff	Government debt portfolio	Cash flow diff	**BOT 12 mth** (par yield)	Cash flow diff	**BOT 6 mth** (par yield)
1.8.83	–100.00	–100.00	0.00	–100.00	0.00	–100.00	0.00	–100.00	0.00	8.33
1.2.84				9.64	9.64	5.14	5.14			7.95
1.8.84	2.85	2.87	0.02	9.29	6.44	13.30	10.45	17.94	15.09	7.12
1.2.85				8.38	8.38	5.07	5.07			6.51
1.8.85	3.18	3.17	–0.01	8.22	5.04	10.98	7.79	15.21	12.03	6.69
1.2.86				7.97	7.97	5.06	5.06			6.55
1.8.86	3.46	3.45	–0.02	7.46	4.00	9.89	6.42	14.00	10.54	5.22
1.2.87				6.50	6.50	4.15	4.15			4.65
1.8.87	3.74	3.66	–0.08	5.74	1.99	7.53	3.79	10.56	6.82	4.90
1.2.88				5.58	5.58	3.30	3.30			4.79
1.8.88	3.92	3.83	–0.09	6.09	2.17	7.62	3.70	9.83	5.91	4.68
1.2.89				5.29	5.29	3.02	3.02			4.85
1.8.89	4.15	4.02	–0.13	5.38	1.23	5.79	1.65	9.85	5.70	5.33
1.2.90				5.60	5.60	3.62	3.62			5.44
1.8.90	4.39	4.28	–0.11	6.13	1.73	7.82	3.42	11.13	6.74	4.89
1.2.91				5.64	5.64	3.77	3.77			5.44
1.8.91	4.72	4.54	–0.17	5.86	1.15	7.27	2.55	10.25	5.53	5.22
1.2.92				5.51	5.51	3.73	3.73			5.04
1.8.92	5.04	4.84	–0.20	5.82	0.78	7.28	2.24	10.41	5.37	6.46
1.2.93				6.13	6.13	4.10	4.10			4.94
1.8.93	216.79	208.76	–8.03	106.79	–110.00	108.28	–108.50	112.72	–104.07	
				Capitalized cash flow values						
	275.63	266.50	–9.14	339.49	63.85	326.22	50.58	312.76	37.13	

of Government securities, we calculated the proportion of CCTs, BTPs, and BOTs in the outstanding stock of debt at the beginning of each year.[4] The Table shows that all three alternatives would have implied an higher cost for the Treasury ranging from a capitalized value of 37 lire for every 100 of principal for the 12 month BOT, to 63.85 lire for the single floater. Relative to the total amount spent by the Government over this 10 year period, the savings were between 12% and 20%. These are sizeable numbers. Of course, one reasonable explanation for such high numbers is an unexpected decline in inflation. The amount of savings was probably exagerated by the illiquidity of the bond, which was placed directly with public sector enterprises and was rarely traded on the secondary market. The placement price may have been artificially high. Nonetheless, it is doubtful that liquidity alone could account for a great part of the savings. Even though *ex post* we cannot assess the role of the inflation risk premium, the mere size of the gains warrant some extra thoughts on the benefits of issuing indexed debt.

4. Real interest rates and expected inflation: lessons from the U.S.

In some other countries, there has recently been increased enthusiasm for issuing indexed bonds, even in countries such as the U.S. where the ratio of Government debt to GDP is not yet a serious impediment to economic policy. Requests for indexed securities now appear frequently in the press: they have been demanded by officials of several federal agencies (*Wall Street Journal*, 24 April 1995), as well as by prominent economists (R. Barro, *Financial Times*, 17 May 1995 and S. Hanke and A. Walters, *Forbes*, 12 September 1994). The strong revival of indexed bonds in the U.S., and perhaps in the world, is partly due to the interest rate dynamics that has characterized U.S. financial markets during the last two years: "... the long bond yield was 8.1 per cent back in April 1992 but then slid all the way to 5.8 per cent during the great bull market in bonds that lasted until late 1993. By November 1994, the yield was back to 8.2 per

[4] At the outset, there were no straight bonds, while they accounted for 29% of the total in 1992. The CCTs chosen in the basket were three issues, which were placed on the market on July 1st, August 1st and September 1st, 1983. The proportion of BOT consisted entirely of 1 year instruments. The BTP chosen was a 5 year issue, placed on the market on August 1st, 1988, maturing the same day as the indexed bond.

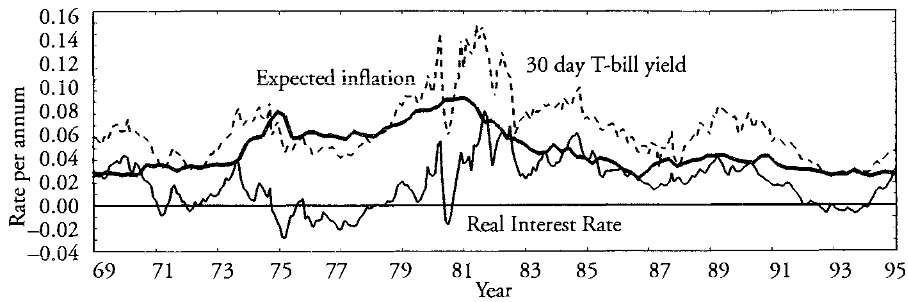
cent. Now, it is testing 7 per cent again. Meanwhile, US inflation has scarcely budged from 3 per cent during all this time" (B. Riley, *Financial Times*, 13 May 1995). If large swings in long-term rates are entirely due to erratic inflationary expectations, index bonds should have an important role in insulating investors' purchasing power from the vagaries of these expectations.

To asses the impact of inflationary expectations and real interest rate movements in U.S. financial markets and, thus, the potential benefits of indexed bonds, we estimated the process for real interest rates and inflationary expectations using a methodology developed by Pennacchi (1991), which is based on the two-factor equilibrium term structure models of Langetieg (1980) and Vasicek (1977).[5] The movements of the real rate and expected inflation were inferred from a time-series cross section of monthly U.S. Treasury bill prices and quarterly survey data on professional forecasters' expectations of the U.S. GNP deflator. Their estimated dynamics clearly indicates that the two variables are instantaneously negatively correlated and that each variable's dynamics depends on the level of the other. The estimates imply speeds of mean reversion for the instantaneous real rate of interest and the instantaneous rate of inflation , measured as their half-lives (the expected time the variables will take to return one-half way back to their steady states following a deviation), equal to 4.5 and 1.8 years, respectively.

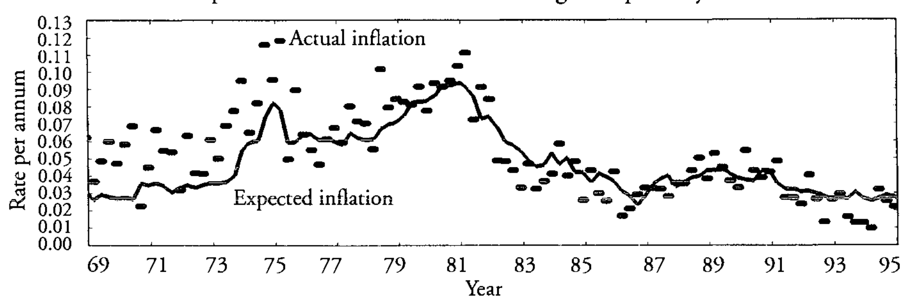
The parameter estimates were also utilized to reconstruct the unobserved time path of the instantaneous real rate and expected inflation. The two estimated series are shown at the top of Figure 4 for the period October 1968 to March 1995. The upward movement in interest rates since the beginning of 1994 is entirely reflected in an increase in real rates, from the extremely low levels (even negative) reached in 1992 and 1993. The figure vindicates the Federal Reserve's claim that real rates were abnormally low and that the rise was meant to bring them closer to a long-run equilibrium level. The bottom of Figure 4 also shows the estimated time series of expected inflation, together with actual inflation which is indicated with dots. It is interesting to note that, unlike the 1970s when expectations appear to consistently underestimate inflation, expectations over the last three years appear to predominantly overestimate actual inflation. The statement that the interest rate increases in 1994 were needed to stabilize expectations, which were running ahead of

[5] The model utilized is explained in the Appendix.

Figure 4. U.S. real rate, expected inflation, and T-bill yield



U.S. expected and actual inflation % changes in quarterly GNP deflator



actual data on prices, seems corroborated by our findings. However, it is doubtful that the real values of indexed bonds would have remained stable during this most recent period since, as can be observed from the top of the figure, real interest rate volatility vastly exceeded that of expected inflation. During the entire sample period, the standard deviation of real interest rates is more than twice that of expected inflation.

The U.S. experience teaches that simple comparisons between current nominal interest rates and actual inflation rates could be misleading. Actual realized inflation and investor inflationary expectations may be quite different. Yet, even if inflationary expectations can be disentangled from actual data, so that we could measure real rates perfectly, we still need to know more in order to determine the desirability of indexed bonds. We would really like to know the risk premium that investors demand in order to hold nominal assets. Measuring this risk premium is not straightforward: the differences between nominal and real term structures reflect not only risk premia but also expected levels of future inflation that can be different over different future horizons. For example, an inflation rate which is expected to revert to a higher long-run mean would imply a difference between nominal and real term structures that increases with maturity, even though investors may demand no premium for inflation risk. Hence, if the higher coupons on nominal bonds are simply compensation for higher expected future inflation, no risk premium is saved by a government issuing indexed bonds. A real bond would simply defer part of the coupon payments. To estimate the size of the risk premium, we need prices on both nominal and indexed bonds, together with a two-factor model of the term structure capable of producing closed form solutions for the risk premia.

5. Indexed bonds and the inflation risk premium in the U.K.

The U.K. is the only country that has issued both conventional and real indexed Government bonds with maturities exceeding 10 years for a significantly long period of time. Thus, we focused on the British experience. As an initial step, we performed an exploratory data analysis to investigate how indexed and conventional bonds differed in terms of risk characteristics. We used weekly returns for a sample of five nominal and five indexed gilts that had relatively similar maturities and that spanned the long end of the term structure (7–20 years). In Table 2 we show that, adjusting for maturity, nominal bonds are more volatile than indexed bonds, though the difference is not substantial: for the typical 10–13 year maturity, the yearly standard deviations of the nominal and indexed gilts are 7.7% and 6.4%, respectively. However, the correlation matrix reveals that the two types of bonds may have different sources of risk. Conventional gilts are highly correlated among themselves, with values ranging from .88 to .97. Indexed gilts are also highly correlated among themselves, with roughly identical correlation coefficients. But the correlations decline to values between .48 and .51 when calculated between similar maturity nominal and indexed gilts, and to a minimum of .32 when the longest nominal is matched with the shortest indexed gilts.

To investigate further the existence and importance of different risk factors, we analyzed the data by assuming a three-factor model of the term structure, where the risk factors are empirically identified by principal component factor loadings of the variance-covariance matrix of returns. Empirical multi-factor models have proved very useful in analyzing and managing interest rate risk. Three factors are typically found: the term structure's parallel shifts, changes in its slope, and what is known as term structure "butterfly" movements[6]. Table 3 shows the loadings for conventional and indexed gilts across maturities. The first factor affects bonds of different maturities in roughly the same way. One can identify it with parallel shifts in the term structures associated with changes in the general level of interest rates. For indexed bonds, however, the first factor accounts for a relatively smaller fraction of returns variability: this factor explains roughly 75% of indexed bonds' returns as opposed to 95% for conventional gilts. The third factor is upward sloping in both indexed and nominal charts. It can thus be identified with movements in the slope of both indexed and nominal term structures. Its contribution to bond return variability is small, roughly 2–3% for both types of bonds. The second factor has a sharply different impact on nominal and indexed bonds: it explains only 5% of conventional gilts returns, but a uniform 23% of indexed bond returns. In addition, the positive sign of the loading indicates that an increase in the risk factor increases indexed bond prices. This factor cannot be easily identified with inflation, since it affects only real bonds. However, if unexpected inflation and real interest rates are instantaneously negatively correlated, as other empirical research has found, then higher inflation may be coincident with lower real interest rates, which would tend to raise indexed bond prices. Whatever the factor identification, the data suggest that there is a significant risk factor typical of indexed bonds.

To assess the risk premia that investors were willing to pay, we again utilized the previously described model of Langetieg (1980) and Pennacchi (1991) where bond prices are driven by two state variables, expected inflation and the real rate of interest. For each month from January 1984 to March 1994, we took prices on a cross section of three long-dated conventional gilts and three long-dated indexed gilts. The composition of the cross section was revised each month to ensure that the maturities of both types of bonds were as similar and constant as possible. The pricing model allows a bond expected rate of return in excess of the instantaneous maturity, risk-free nominal interest rate to be expressed as the sum of the bond premium due to inflation risk and its premium due to real interest rate risk. Each risk premium, in turn, is the product of the given factor's market price of risk multiplied by the quantity of that factor's risk contained in the stochastic component of the bond's rate of return.

In principle, the parameters of the stochastic process driving the two

[6] See Litterman and Scheinkman (1991).

Table 2. Correlation matrix if weekly log price changes (1/1987–3/1994)

Bond	Years to maturity	St. dev.	Nominal 07–10	Nominal 10–13	Nominal 14–16	Nominal 16–18	Nominal 18–20	Indexed 07–10	Indexed 10–13	Indexed 14–16	Indexed 16–18	Indexed 18–20
			0.066555	0.077613	0.088136	0.093208	0.094586	0.052578	0.064445	0.067473	0.070079	0.071788
Nominal	07–10	0.066555	1	0.967942	0.903022	0.890268	0.878794	0.516888	0.582183	0.591473	0.587952	0.598926
Nominal	10–13	0.077613	0.967942	1	0.963046	0.953469	0.944804	0.455841	0.539284	0.553423	0.55474	0.566836
Nominal	14–16	0.088136	0.903022	0.963046	1	0.987778	0.986177	0.35909	0.453227	0.476091	0.483144	0.498651
Nominal	16–18	0.093208	0.890268	0.953469	0.987778	1	0.991265	0.348348	0.444859	0.467411	0.477032	0.492245
Nominal	18–20	0.094586	0.878794	0.944804	0.986177	0.991265	1	0.329358	0.43155	0.45444	0.46359	0.482177
Indexed	07–10	0.052578	0.516888	0.455841	0.35909	0.348348	0.329358	1	0.924019	0.902749	0.880941	0.868433
Indexed	10–13	0.064445	0.582183	0.539284	0.453227	0.444859	0.43155	0.924019	1	0.988477	0.972404	0.965708
Indexed	14–16	0.067473	0.591473	0.553423	0.476091	0.467411	0.45444	0.902749	0.988477	1	0.988819	0.983504
Indexed	16–18	0.070079	0.587952	0.55474	0.483144	0.477032	0.46359	0.880941	0.972404	0.988819	1	0.992152
Indexed	18–20	0.071788	0.598926	0.566836	0.498651	0.492245	0.482177	0.868433	0.965708	0.983504	0.992152	1

Table 3. First, second, and third principal components of weekly log price changes

Bond	Years to maturity	Component (% explained)						Cumulative % explained
		e1		e2		e3		
Nominal	07–10	–1.65527	(95.07130)	–0.16802	(0.97953)	–0.32179	(3.59289)	99.6437
Nominal	10–13	–1.97507	(96.35782)	–0.31505	(2.45176)	–0.19280	(0.91818)	99.7278
Nominal	14–16	–2.19767	(94.52356))	–0.51556	(5.20211)	0.05499	(0.05919)	99.7849
Nominal	16–18	–2.31384	(94.08840)	–0.55875	(5.48654)	0.11457	(0.23066)	99.8056
Nominal	18–20	–2.32950	(93.47722)	–0.58771	(5.94987)	0.15569	(0.41756)	99.8446
Indexed	07–10	–0.89170	(64.89185)	0.60670	(30.04010)	–0.14676	(1.75787)	96.6898
Indexed	10–13	–1.26028	(74.27629)	0.73393	(25.18979)	–0.01628	(0.01239)	99.4785
Indexed	14–16	–1.35121	(75.99249)	0.75411	(23.66997)	0.03557	(0.05265)	99.7151
Indexed	16–18	–1.40984	(76.55287)	0.77079	(22.88201)	0.08432	(0.27384)	99.7087
Indexed	18–20	–1.46411	(77.84949)	0.76750	(21.39247)	0.10193	(0.37736)	99.6193

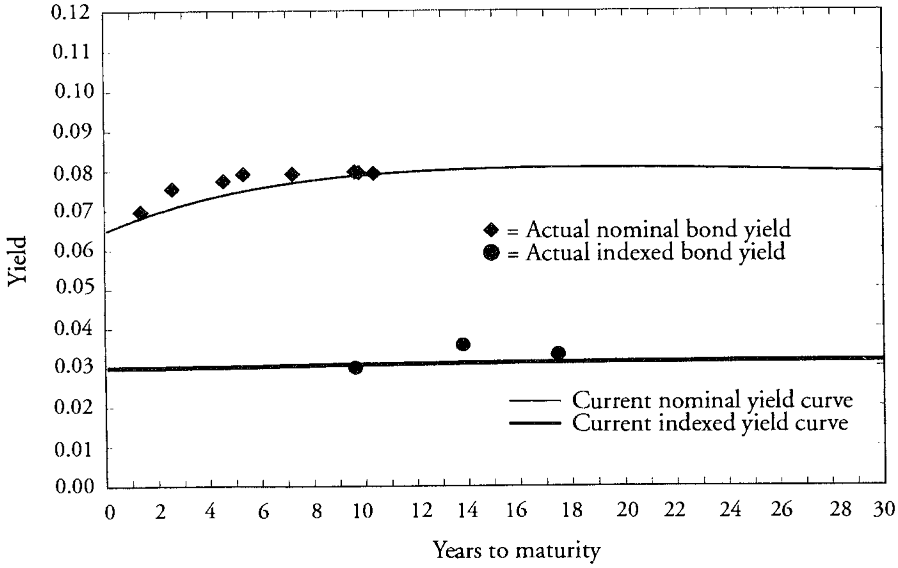
state variables could be estimated jointly with the market prices of risk; in practice, small sample estimation problems make simultaneous identification of all of these parameters quite difficult. To reduce the number of parameters to be estimated, we took a somewhat Bayesian approach by presetting certain parameters at values that appear reasonable based on historical experience or prior empirical work. First, we set the steady state values of the real rate and inflation equal to 3 per cent and 3.5 per cent, respectively. Second, the parameters other than the market risk premia describing the dynamics of the real rate and expected inflation were set at the estimates obtained using U.S. data, which we described earlier. Although this will produce real interest rate and expected inflation dynamics that are specific to the U.S., Kandel, Ofer, and Sarig (1995) found qualitatively similar dynamics for Israeli real interest rates and inflation using data on nominal and indexed bonds issued by the Government of Israel. Having fixed this subset of parameter values, we then estimated the market price of inflation and real rate risk using the time-series cross-section data on U.K. gilt prices. While some of our parametric assumptions were admittedly crude, we believe that the results should be evaluated against the capability of the model to fit the existing U.K. term structure.[7]

Surprisingly, the model does quite well. Figure 5 reports the estimated nominal and indexed yield curves for the U.K., assuming the current values for expected inflation and the real rate are equal to their assumed steady state values, a reasonable assumption given that the current actual level of inflation is roughly 3% and the short-term nominal rate is approximately 6.5%. Also plotted against their durations are the May 30, 1995 yields to maturity of the 2, 3, 5, 7, 10, 15, 20 and 30 year benchmark conventional gilts and the real yields on indexed gilts maturing in 2006, 2013, and 2020[8]. It is important to stress that these yields were not used to estimate the parameters, and thus the fit is out of sample. Given this, and the manner in which many of the parameters were obtained, the model appears to replicate market prices in a satisfactory way. The indexed bond yield curve is flat because the estimated market price of real interest rate risk was found to be close to zero. In contrast, the estimated

[7] We do not account for taxes which, however, have been shown to play a significant role (Woodward, 1990).

[8] To calculate the real yields on indexed bonds, we subtracted the accrued inflation since the issue date from the quoted price of the gilt, the coupons to be paid and the principal amount. As for conventional gilts, the yields were plotted against the bond durations. Data on both conventional benchmark gilts and indexed gilts were taken from Datastream.

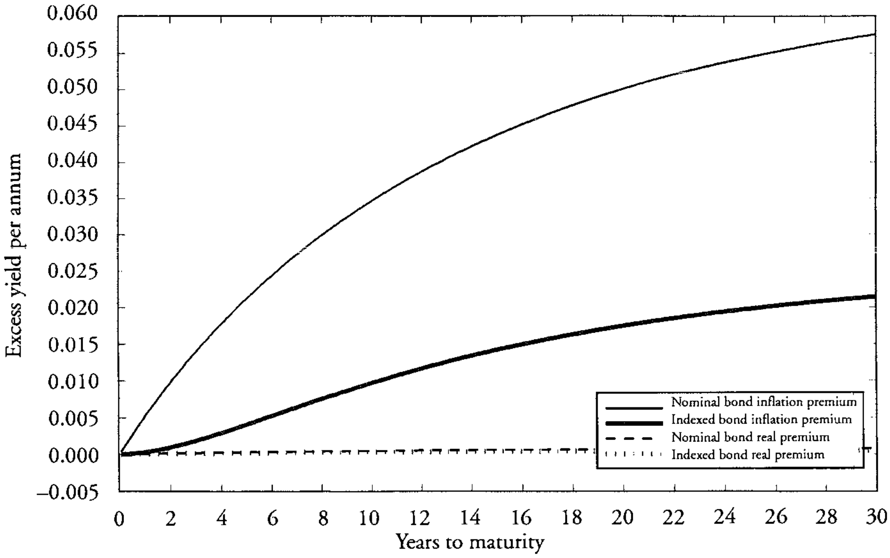
Figure 5. U.K. nominal and indexed bond term structures (current equals steady-state)



market price of inflation risk was large and significant which, in spite of the fact that current real rates and expected inflation were assumed to equal their steady state values, resulted in an upward sloping nominal term structure.

In Figure 6, we graphed the estimated inflation and real risk premia as a function of bond duration. Notice that even indexed bonds pay some premium for inflation risk because of the dynamic relationship between the two state variables. Based on the positive estimates of the state variables' impact on each other's dynamics, an unexpected increase in current expected inflation (given no instantaneous movement in the real interest rate) will increase the likelihood of higher future short-term real interest rates, which will decrease the current price of real bonds. Thus, investors in real bonds will demand a premium to hedge against this (indirect) inflation risk. The difference between the two inflation risk premium curves measures the potential savings for the Government from real indexed bonds, as opposed to conventional straight bonds. For example for a twenty year debt instrument, savings are approximately 300 basis points, a substantial amount, albeit smaller than what is typically discussed informally. Note that since the estimated market price of real interest rate risk is approximately zero, both bonds' real interest rate risk premium curves are flat at zero. Thus, there is no relative benefit to either bond along this dimension.

Figure 6. U.K. nominal and indexed bonds: real and inflation risk premia



6. Indexed bonds and the inflation risk premium in Sweden

In the Spring of 1994, the Kingdom of Sweden issued zero coupon Treasury bonds with principal tied to the cost-of-living index and maturities of 10 and 20 years. Because the two bonds do not pay coupons, their yield to maturity exactly measures the return to deferring a unit of consumption into the future. In addition to long maturity indexed bonds, Sweden has issued nominal Treasury bonds for several maturities. In principle, the contemporaneous observation of real and nominal term structures could allow for estimates of risk premia for inflation and real interest rate risk. As with British gilts, we used a two-factor term structure. The multiple parameter estimation problems encountered for the case of the U.K. is aggravated in Sweden by the lack of sufficiently long time series on indexed bond prices. Only one year of prices precludes any attempt to estimate all of the model's parameters simultaneously, especially those relating to the dynamic behavior of real interest rates and inflation, which clearly requires observations from different economic cycles.

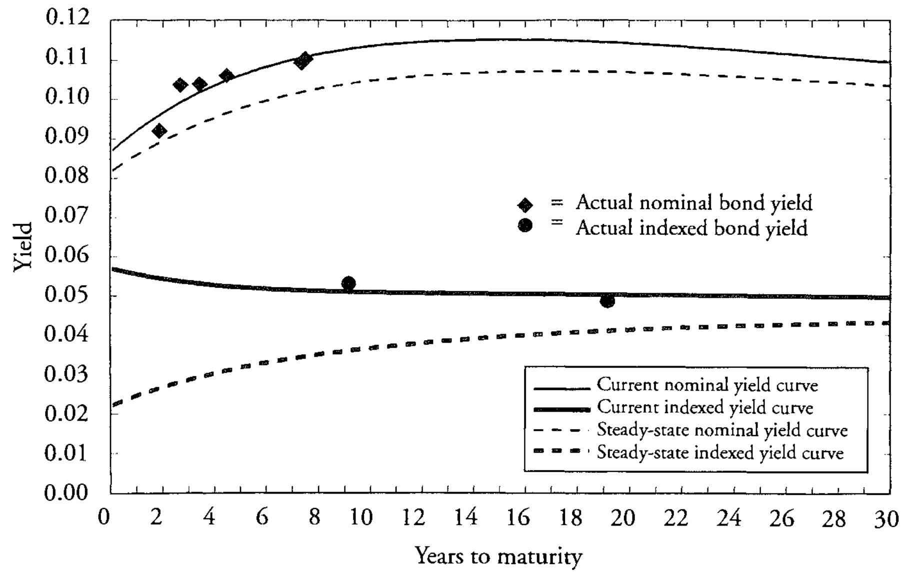
Given these difficulties, the following exercise should be viewed as no more than a sensible starting point for measuring the price of inflation and real interest rate risk in the market for Swedish Government bonds. In estimating these risk premia, we used prices from a cross section of nominal and indexed bonds on January 30, 1995. We chose only this one day because it was the date of the latest auction of both 10 and 20 year indexed bonds and because demand exceeded supply by a factor of 3, making the auction price significant. This day was prior to the European exchange rate crisis – an abnormal event which might have introduced significant noise into market prices. The approach we followed was similar to that taken in our estimation of risk premia for the U.K. in that we preset some of the potentially estimable parameters at values we regarded as reasonable. As with the U.K., we set the matrix determining the dynamics of real interest rates and inflation (matrix B of equation (A2)) at the estimates obtained using U.S. data. Again, while there is no guarantee that Swedish real interest rates and inflation interact in a similar manner, this assumption should be evaluated against the capability of fitting both the real and nominal term structure in Sweden. However, because Sweden has experienced relatively high volatility for interest rates and inflation, we estimated the covariance matrix for real rates and inflation (matrix S of equation (A2)) using Swedish data on inflation and changes in real GDP (a proxy for real interest rates) over the last 25 years.

We obtained estimates for the annualized standard deviations of the change in real rates of 3.2% and of the change in inflation of 2.2%, with a correlation of –.237. Using these same data, we then made two sets of conjectures about the current levels and long-run means of both the instantaneous expected inflation rate and the real rate of interest. While output growth averaged 2.8% over the last 25 years, a reasonable hypothesis is that the current burden of adjusting fiscal imbalances will reduce the potential growth rate of the Swedish economy in the long run. Thus, we set the steady state real interest rate at 2.2%. Short-term interest rates above 8% at the end of January, combined with an inflation rate of 2.6%, suggest that the unobservable instantaneous real rate was above 5%. For the simulation we chose 5.7%, implying that the real rate of interest will be expected to decline to its long-run value. Such a dynamic was implied by the market yield curve for real rates observed at the January 30, 1995 auction, which was negatively sloped for very long maturities, between 9.16 and 19.16 years. As for expected inflation, we considered two different scenarios. The first assumes a level of 3%, slightly above the current inflation rate of 2.6%, and a long-run level of 3.5%, which is roughly consistent with a stable exchange rate *vis-à-vis* the Deutsche Mark if the inflation rate in Germany is expected to remain around 2.5%. The second scenario is more pessimistic and assumes that the steady state inflation rate will be 6%, a level which is close to the average inflation over the last 25 years.

Given these parametric assumptions, we then used the January 30, 1995 prices of both nominal and indexed bonds to estimate the prices of inflation and real interest rate risk. Indeed, we could observe only two points on the real curve, a rate of 5.316% for the indexed bond with a residual life of 9.166 years and 4.87% for the bond with 19.166 years to maturity. For the nominal yield curve, we could observe the yield to maturity on the 2, 3, 5, 7, 10, and 15 year Treasury bonds, classified as benchmark bonds according to Datastream. Because the nominal bonds pay coupons, we approximated the term structure by associating those yields to the benchmark durations, of 1.87, 2.66, 3.41, 4.47, 7.34 and 7.49 years, respectively. Since we had only two points for the indexed bonds, as opposed to six for the nominal, we assigned a triple weight to the indexed yields in the non- linear optimization algorithm that we used to minimize the sum of squared deviations between these actual bond yields and the theoretical model bond yields.

Employing this optimization procedure, we obtained two sets of estimates of the market price of inflation and real rate risk, one for the low steady state inflation scenario and one for the high state scenario. Using these risk premia estimates, we then graphed the theoretical nominal and indexed term structures under both inflation scenarios. In spite of the rough parameter settings that we used, Figure 7 shows that the model can fit the actual yields of nominal and indexed bonds, denoted by diamonds and dots in the figure, rather well. To help interpret these current nominal and indexed term structures, we also graphed corresponding term structures setting the initial values of the real rate and expected inflation at their steady states, rather than their assumed current values. These steady-state term structures give a better indication of the effects of risk premia since their shapes are not affected by expected movements of the state variables. Returning to the current nominal and indexed yield curves, we see that the fit is roughly equivalent under both inflation scenarios: when the inflation rate is assumed to revert to a higher value, much of the level and curvature of the nominal curve is accounted for by the rise in expected inflation, while that of the real rate appears unaffected by the inflationary scenario, as one would expect.

Figure 7. Swedish nominal and indexed bond term structures (steady-state inflation = 6.0%)



We argued earlier that if the higher interest rates on nominal bonds are exclusively due to higher expected inflation, a government would save very little by issuing real indexed bonds, providing that investors' expectations are correct, on average. Figures 8 and 9 display the two premia for both nominal and indexed bonds by maturity, for both the low and the high steady state inflation scenario. In the low inflation scenario, the savings for the Government from issuing indexed bonds appears quite large. As an example, for the 10 year maturity, a typical benchmark maturity for treasuries in European countries, the difference between the inflation premia on nominal and indexed bonds is close to 400 basis points, and becomes 500 when the maturity moves to 20 years. The benefits, however, decline to only 130 and 160 basis points, respectively, if the market expects inflation to settle down at a much higher level in the long run. This might appear to be a contradiction, since greater inflation would seem to reflect greater inflation risk, and hence greater savings. The explanation, however, is that the shape of the term structure might be a reflection of expected movements in the state variables (and small risk premia) or large risk premia (and small expected movements in the state variables): to the extent that expectations explain a greater proportion of market prices, risk premia must explain a lower fraction. Thus, we cannot

Figure 8. Swedish nominal and indexed bonds: real and inflation risk premia (steady-state inflation = 3.5%)

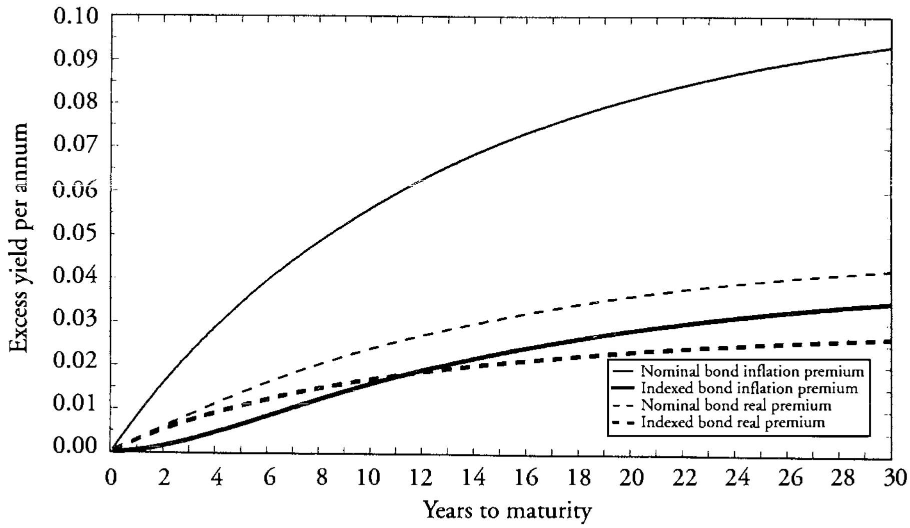
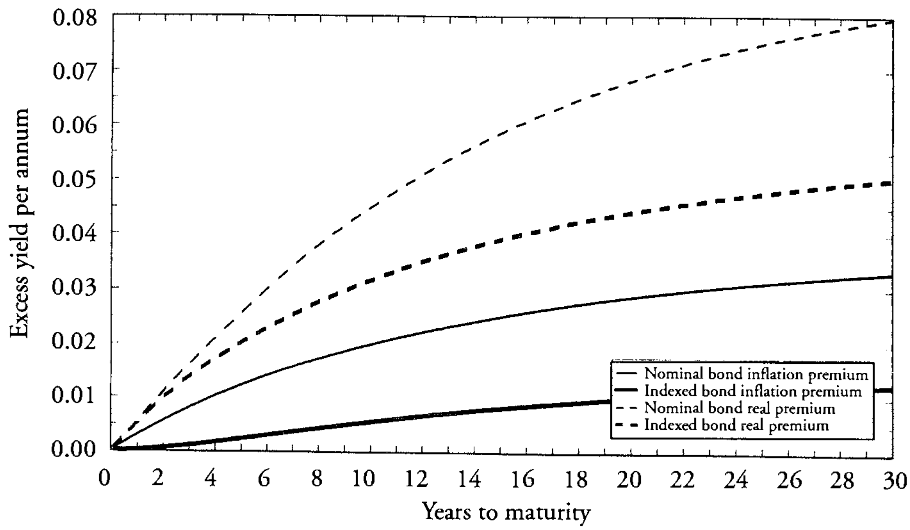


Figure 9. Swedish nominal and indexed bonds: real and inflation risk premia (steady-state inflation = 6.0%)



state with a high degree of certainty whether government savings are small or large, since it depends on investors' long-run expectations.

7. What lessons have we learned?

We looked at a number of financial markets to see whether the pricing of government debt can suggest how high-debt countries should manage their liabilities. Lesson number one is that floaters do not provide substantial savings. While, in theory, they might reduce the risk of rationing, there is no evidence that they command a lower default risk premium than other forms of debt. In addition, investors seem unwilling to pay a premium relative to fixed coupon bonds for the better inflation hedge provided by floaters. Lesson number two is that financial innovation is another unpromising avenue to government debt management. Attempts to make financial markets more complete may often receive an unenthusiastic response by investors. While, in theory, governments could save money by issuing securities that provide previously unavailable protection against risks, in practice, it is very difficult to gauge investor demand for such insurance. In addition, markets seem to reward financial instruments that can be stripped into simple, tradable, and easily hedged components. Government debt with embedded contingent claims that does not satisfy these requirements may be priced at far less than its fair probabilistic value. Governments would do best to leave financial innovation to the private sector and focus debt policies on making their bond issues as liquid as possible. Governments might introduce new financial instruments only after there have been similar innovations in the private sector and only when the treasury is likely to be a lower cost supplier.

Lesson number three is that financial markets are willing to pay a premium for depth and liquidity. Cost savings can be achieved by ensuring that single bond issues reach a size which is considered large by global investors. Because size is a necessary, but not a sufficient, condition for liquidity, debt management policy should promote a market micro-structure that concentrates trading on a few benchmark bonds. In addition, the development of derivatives with risk characteristics similar to the benchmarks should be encouraged to enable both domestic and foreign market participants to easily transfer risk. The Italian experience on this score is very convincing. Single bond issues are re-opened several times, until they reach a size of between 10,000 and 30,000 billion lire (equivalent to 6–20 billion dollars). Special Treasury bond dealers are committed to making two way prices at any time via an automated electronic market. In addition, futures contracts on Italian Government bonds, traded in both London and Milan, allow dealers to hedge inventories and promote market efficiency by keeping cash and futures prices in line. Newly issued, on-the-run, 10 year straight bonds, which have the status of benchmark issues and are typically deliverable against future contracts, usually trade at a considerable premium relative to the term structure under very different market conditions: on average roughly 50 basis points relative to bonds with equivalent risk characteristics.

Lesson number four is that real indexed bonds can reduce the cost of debt, but the savings are hard to reckon, and may be overestimated by a simple analysis of the term structure. A steep nominal term structure, along with a lower and flat real term structure, may not reflect a significant inflation risk premium if investors perceive current inflation and/or real interest rates to be different from their steady state levels. In addition, one cannot rule out that indexed bonds may also require an inflation risk premium when changes in expected inflation and real interest rates are dependent. In most cases, however, we do find that nominal bonds require an inflation risk premium relative to indexed bonds which translates into potential savings for the treasury of up to several hundred basis points. A strong case can be made for increased issuance of indexed bonds.

Appendix

In this appendix we describe the two factor bond price model that underlies our empirical estimates of the inflation rate risk premium. Defining the price index on date t as $I(t)$, the actual inflation rate is assumed to follow the stochastic process:

$$\frac{dI}{I} = \pi(t)\,dt + \sigma_I dz_I \tag{A1}$$

where $\pi(t)$ is the instantaneous expected rate of inflation at date t, σ_I is the annualized instantaneous standard deviation of inflation, and dz_I is a standard Wiener process. The price of a nominal bond is assumed to be determined by two risk factors: the current instantaneous expected rate of inflation and the current short-term (instantaneous maturity) real interest rate, $r(t)$. We allow these two factors to be correlated, consistent with the findings of numerous empirical studies. The stochastic evolution of the two risk factors is described as:

$$\begin{bmatrix} dr \\ d\pi \end{bmatrix} = \left(\begin{bmatrix} a_r \\ a_\pi \end{bmatrix} + \begin{bmatrix} b_r & b_{r\pi} \\ b_{\pi r} & b_\pi \end{bmatrix} \cdot \begin{bmatrix} r(t) \\ \pi(t) \end{bmatrix} \right) \cdot dt + \begin{bmatrix} \sigma_r \, dz_r \\ \sigma_\pi \, dz_\pi \end{bmatrix} \qquad \text{(A2)}$$

The elements of the A vector determine the steady state levels of the two factors while the elements of the B matrix determine the factors' dynamics and speeds of adjustment to these steady state levels. The annualized instantaneous standard deviations of changes in the real interest rate and the expected rate of inflation are given by σ_r and σ_π, respectively. An attractive feature of this model is that the two factors, $r(t)$ and $p(t)$, may be instantaneously correlated, i.e., $\rho_{r\pi} \neq 0$ and have an impact on each other's dynamics. A limitation is the constant volatility assumption for the state variables.

Let the current (date t) price of a zero coupon bond with a time until maturity of T be denoted $P(T)$. Also, let $m(t)$ denote the bond's expected rate of return in excess of the instantaneous maturity, risk-free nominal interest rate. This excess rate of return can be expressed as the sum of the bond's premium due to inflation risk and its premium due to real interest rate risk. Each risk premium, in turn, is the product of the given factor's market price of risk, Φ, multiplied by the quantity of that factor's risk contained in the stochastic component of the bond's rate of return:

$$m(t) = \Phi_r \frac{P_r}{P} \sigma_r + \Phi_\pi \frac{P_r}{P} \sigma_\pi \qquad \text{(A3)}$$

Given the above assumptions, the price of a nominal or real zero coupon bond takes the form:

$$P(T) = K(T) e^{-D(T) s(t)} \qquad \text{(A4)}$$

where $s(t) = (r(t)\ p(t))'$ is a vector of the date t values of the two unobserved state variables, $D(T)$ is a 1x2 vector, and $K(T)$ is a constant. A similar functional form can be derived for the expected level of inflation over any finite future period. Using these formulas, the parameters of the B matrix, σ_r and σ_π and $\rho_{r\pi}$ were estimated by maximum likelihood using a Kalman filter to compute the likelihood function. The details are given in Pennacchi (1991).

References

Alesina, A., A. Prati and G. Tabellini (1990), Public Confidence and Debt Management: A Model and a Case Study of Italy, in: R. Dornbusch and M. Draghi, eds., Public Debt Management: Theory and History (Cambridge University Press, Cambridge) 238–260.

Bohn, H. (1988), Tax Smoothing with Financial Instruments, Working Paper 27-1988, Rodney L. White Center for Financial Research, University of Pennsylvania.

Cox, J., J. Ingersoll and S. Ross (1985), A Theory of the Term Structure of Interest Rates, Econometrica 53, 363–384.

El-Karoui, N. and H. Geman (1992), The Valuation of General Floating-Rate Notes and Interest Rate Swaps: A Probabilistic Approach, Working Paper, University of Paris and ESSEC.

Fischer, S. (1983), The Welfare Aspects of Government Issue of Indexed Bonds, in: R. Dornbusch and M. Simonsen, eds., Inflation, Debt and Indexation (MIT Press, Cambridge, MA), 223–246.

Gale, D. (1994), The Efficient Design of Public Debt, in: D. Gale and F. Allen, Financial Innovation and Risk Sharing (MIT Press, Cambridge, MA), 271–307.

Gorton, G. and G. Pennacchi (1991), Financial Intermediaries and Liquidity Creation, Journal of Finance 45, 49–72.

Kandel, S., A. Ofer and O. Sarig (1992), Real Interest Rates and Inflation: An Ex-Ante Empirical Analysis, Working Paper, Tel-Aviv University.

Langetieg, T. (1980), A Multivariate Model of the Term Structure, Journal of Finance 35, 71–97.

Litterman, R. and J. Scheinkman (1991), Common Factors Affecting Bond Returns, Journal of Fixed Income, June, 54–61.

Peled, D. (1985), Stochastic Inflation and Government Provision of Indexed Bonds, Journal of Monetary Economics 15, 291–308.

Pennacchi, G. (1991), Identifying the Dynamics of Real Interest Rates and Inflation: Evidence Using Survey Data, Review of Financial Studies 4, 53–86.

Vasicek, O. (1977), An Equilibrium Characterization of the Term Structure, Journal of Financial Economics 5, 177–188.

Woodward, T. (1980), The Real Thing: A Dynamic Profile of the Term Structure of Real Interest Rates and Inflation Expectations in the United Kingdom, 1982-89, Journal of Business 63, 373–398.